



HOME SCHOOLING MATERIAL

PASS A LEVEL

**CHEMISTRY, MATHEMATICS,
ENTREPRENEURSHIP**



YOUR GUIDE AWAY FROM SCHOOL

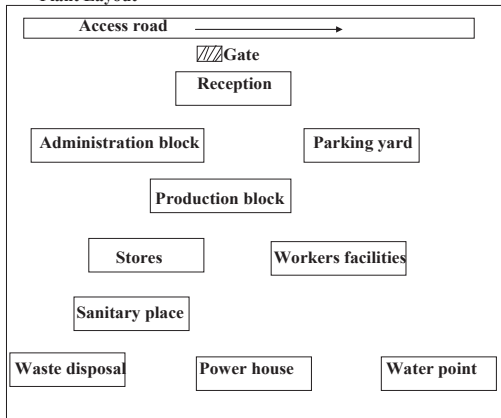
ENTREPRENEURSHIP EDUCATION SET ONE ANSWERS

1. (a) A purchase order form has the following contents:

- ▷ Date
- ▷ Address of the supplier (name & address)
- ▷ Inside address
- ▷ Document title "purchase order"
- ▷ Purchase order number
- ▷ Breakdown of machinery being ordered for: number,
- ▷ Description, quantity, unit cost, amount and grand total
- ▷ Delivery date
- ▷ Suggested type of packaging

- ▷ Person making order
- ▷ Frame
- (b) Contents of the plant layout
- ▷ Access road with an arrow
- ▷ Reception
- ▷ Parking yard
- ▷ Administration block
- ▷ Production block
- ▷ Workers facilities
- ▷ Sanitary place
- ▷ Stores
- ▷ Water point
- ▷ Power house
- ▷ Waste disposal
- ▷ Frame

Plant Layout



(c) A marketing plan
▷ Name and address with statement "marketing plan"
▷ Target customers (mentioned)

- ▷ Products to be offered; i.e., maize flour and quantities
- ▷ Position of competitors (mentioned and their location)
- ▷ Pricing and sales strategy; i.e.

Pricing structure

Products sold	Our price	Price of competitor kg	Difference

Projected sales

Product	Quality kg	Price (shs)	Amount (shs)

- ▷ Promotion and advertising; the objectives to be achieved by promotion and advertising
- ▷ To inform target customers about the availability of the product
- ▷ To increase sales and profits
- ▷ To compete with other businesses

- ▷ Distribution strategy; distributing directly to customers, use of retail outlets/supermarkets at the factory premises, use of retailers and wholesalers in different parts of the country.
- ▷ Projected marketing expenses in a table form

Promotional method	Period	Quantity	Unit cost (shs)	Total for the month (shs)	Total for the year (shs)
Sign post	Per year				
Business cards	Per year				
Brochures	Per year				
Sales person	Per month				
News advert	Per month				
Total marketing expenses					

THE TEACHERS



KEDRETH ASIMWE,
MBARARA HIGH SCHOOL



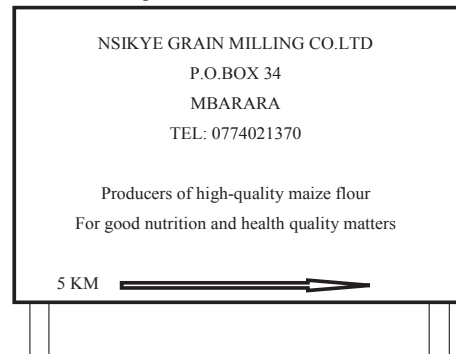
ALLAN ANABWE BUHAMIZO,
NTARE SCHOOL

(d) An effective advert has the following contents

- ▷ Name of the advertising medium and signpost
- ▷ Name and address with postal and telephone of the business
- ▷ Products being offered
- ▷ Benefits of the products to

the customer

- ▷ An appealing phrase/message
- ▷ Arrow showing the direction of the business with (km)
- ▷ Stands
- Note:** Avoid congesting the advert



2(a) A letter to the district engineer seeking technical advice on the proposed expansion

Contents of the letter

- ▷ Name and address of the business reflecting the project that is being produced
- ▷ Date
- ▷ Reference
- ▷ Inside address
- ▷ Salutation; i.e., Dear sir

- ▷ Subject heading
- ▷ Body (containing the reason, time and venue)
- ▷ Complimentary close
- ▷ Signatory (sign, name and title)
- ▷ Carbon copy
- ▷ Enclosure

(b) Contents of the financial plan

(i) Name and address of the business with statement "financial plan"

Estimated fixed capital requirements of the business

ITEM	AMOUNTS (SHS)
Fixed capital expenses	
Acquisition of land	XX
Construction of building	XX
Purchase of machinery and equipment	XX
Purchase of tools	XX
TOTAL FIXED CAPITAL	XXX

(ii) Estimated working capital requirement

Working capital expenses	AMOUNTS (SHS)
Raw materials	XX
Power and utilities	XX
Packaging materials	XX
Direct labour	XX
Administrative expenses	XX
Marketing expenses	XX
TOTAL	XXX

(iii) Expected sources of funds.

SOURCE OF FUNDS	AMOUNTS (SHS)	COST IMPLICATIONS
Own funds	XX	
Family and friends	XX	
Trade credit	XX	
Loan	XX	
TOTAL		

(IV) Estimated sales per year shs XXX

(V). Expected profit of the business per year

DETAILS	AMOUNT (SHS)
Sales	XX
Less: Estimated products	
Expenses	XX
Gross profit	
Less: administrative over needs	
Tax to be	XX
Next profit	XXS

(VI) cash flow projections of the business; i.e.,
Cash inflow – Cash outflow = Net cash

(VII) Breakeven scale of the business: the expected breakeven scale of the business is where total cost is equal to total revenue

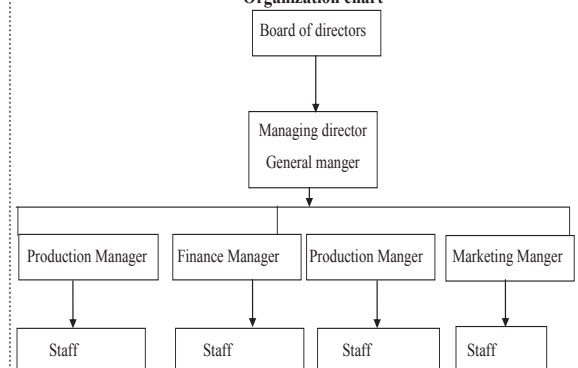
(c) Operational budget to finance the expansion of the cooking oil business.

Name and address with the word operational budget.

DETAILS	SHS	SHS
Projected sales revenue		XXX
Less: expected cost of sale		
Labour or wages	XX	
Raw material	XX	
Packaging	XX	
Total cost		XX
Expected gross, profit		XXX
Less: operating expenses		
Utilities (i.e electricity, water)	XX	
Insurance	XX	
Salaries	XX	
Total expenses		XX
Net profit		XXX

(d) Organizational chart for expanded cooking oil business.
-Name and address of the business with the statement

Organization chart



Turn to next page

ENTREPRENEURSHIP EDUCATION SET ONE ANSWERS

3. You plan to start a business based on the environment in your community.

- (a) Factors considered when formulating the business idea other than market include;
- Return on the investment/profitability
 - Level of completion or ease entry into the market
 - Availability of required raw materials
 - Availability of market
 - Cost of the business venture
 - Availability of required raw materials
 - Availability of labour
 - Availability of required technology
 - Availability of land
 - Strategic business location
 - Business risks involved
 - Availability of infrastructure and other services
 - Personal consideration or interest
 - Acceptability in the community
 - Government policy or support to the business
 - Nature of the customers

(b) Personal entrepreneurial skills to make your business

- Planning skills
- Leadership
- Marketing skills
- Organisation skills/co-ordinating skills
- Time management skills
- Communication skills
- Controlling skills
- Budgeting skills
- Decision making skills
- Delegation skills
- Negotiating skills
- Purchasing skills
- Interpersonal skills
- Technical skills
- Net working
- Innovation and creativity skills
- Coping with change skills

(c) Social concerns to be met by the business

- Unemployment
- Poverty
- Natural disasters like floods and epidemics like coronavirus (COVID-19)
- Environment degradation
- School dropout
- Drug abuse
- Unsafe water and lack of safe water
- Inadequate health education
- Disadvantages groups like women, people with disabilities and orphans
- Inadequate supply of goods and services
- Insecurity both political and social
- Bad foreign influence; e.g., gayism/moral decay
- Domestic violence
- Inadequate market for products

(d) Aspects to be addressed during business implementation planning It involves using the contents of the action plan which include;

- Resources needed in implementation of the planned activities
- Time frame; i.e., how long each activity is to take
- Persons in charge – workers responsibilities
- Indicators of success
- Remarks – regards to work done
- Activities to be carried out

4. (a)

BULA ENTERPRISES LTD'S CASH FLOW STATEMENT FOR THE MONTHS JAN – APRIL 2019

DETAILS	JAN	FEB	MARCH	APRIL
Cash in flow	shs	shs	shs	shs
Opening balance.	20,000,000	(2200,000)	(26640,000)	(82,194,500)
cash sales	8,000,000	8,400,000	8,820,000	9261,000
Receipt from debtors	5,500,000	5,500,000	5,500,000	5500,000
cash donation				10,000,000
Sale of old vehicle			9,000,000	
Total cash inflow	335,500,000	11,700,000=	(3,320,000)	57,433,500
Less: cash outflow				
Purchase of machine			9,750,000	
Import duty			487,500	
Salaries & wages	30,000,000	30,000,000	60,000,000	
Machines servicing	300,000	300,000	300,000	300,000
Loan principal repayment		2000,000	2000,000	2000,000
Loan interest payment		100,000	100,000	100,000
Purchase of raw materials	5,400,000	5,940,000	6,237,000	6,548,850
Total cash outflow	35,700,000	3,8340,000	78,874,500	8,948,850
Net cash position	2,200,000	2,6640,000	82,194,500	66,382,350

(b) Personal entrepreneurial skills to make your business

6 (i) gross income

Gross income = business income + employment

Income + property income – exempt income

= 8,000,000 + 19,200,000 + 4,000,000 – 1,200,000

Shs 30,000,000

(ii) Chargeable income

Chargeable income = gross income – expenditure income

= 30,000,000 – 9,600,000

Shs 20,400,000

(iii). Tax liability (tax payable)

Tax liability = $300,000 + 30 \times \frac{(20,400,000 - 4,920,000)}{100}$

= 300,000 + 4,644,000

Shs 4,944,000

5(a)(i) Gross profit margin = $\frac{\text{gross profit}}{\text{Net sales}} \times 100$

Net sales

But gross profit = Net sales – cost of sales
= 288,000,000 – 201,600,000
= Shs 8,6400,000
= **30%**

(ii) Stock turn = $\frac{\text{cost of sales}}{\text{Average stock}}$

But average stock = $\frac{\text{opening stock} + \text{closing stock}}{2}$
= $\frac{40200,000 + 5400,000}{2}$
= **shs 45,300,000**

Stock turn over = $\frac{201600,000}{45300,000}$
= **4.5 times**

(iii) Debtors collection period
= $\frac{\text{debtors} \times \text{number of days in a year}}{\text{Net sales}}$
= $\frac{90,000,000 \times 366}{288,000,000}$
= **114 days**

(iv) Leverage ratio / gearing ratio = $\frac{\text{longterm liabilities}}{\text{Owner's equity}}$
= $\frac{38500,000}{68400,000}$
1:0.56 or (56.3%)

(v). Interest cover = $\frac{\text{net profit before tax and interest}}{\text{Interest expense for the year}}$

Interest expense for the year
= $\frac{40,600,000}{5,700,000}$
= **7.12**

Working capital /current ratio = $\frac{\text{Current assets}}{\text{Current liabilities}}$

Current liabilities
= $\frac{36100,000}{16800,000}$
= **2.15:1**

(vii) Net profit margin = $\frac{\text{net profit} \times 100}{\text{Net sales}}$

But net profit before interest – interest expenses for the year
= $\frac{40600,000 - 5,700,000 \times 100}{288,000,000}$
= **12.11%**

Interpretation

(b)(i). Stock turn over: 4.45 or 4 times

The business on average replaced its stock 4.45 or 4 times in a year

(ii) Interest cover. One quarter of the net profit made by the business before interest and tax payment is paid out as interest.

ENTREPRENEURSHIP QUESTIONS (AENT002)

SECTION A (20 marks)

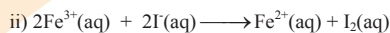
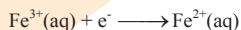
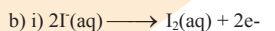
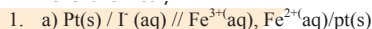
- 1 (a) i) Distinguish between paid employment and self-employment. (2 marks)
 (ii) Mention two reasons self-employment may not easily be attained in Uganda. (2 marks)
 (b) List four hindrances faced by the government when promoting small and medium enterprises in Uganda. (4 marks)
 (c) (i) Give two tools used in proper and efficient time management. (1 mark)
 (ii) Mention two methods of time wastage in business. (2 marks)
 (d).i) List four techniques of handling customers' objections. (4 marks)
 (e). (i) Differentiate between a feasible business and a viable business. (2 marks)
 (ii) Give two characteristics of a viable business. (2 marks)

SECTION B (80 marks)

- 2 (a) Justify the need for adopting change in business (10 marks)
 (b) Explain the factors that limit entrepreneurs from adopting change. (10 marks)
 3 (a) How can an entrepreneur manage competition in market. (10 marks)
 (b) Discuss the effect of business struggle for customers. (10 marks)
 4(a) Explain the components of entrepreneurial environment that influence business intentions. (10 marks)
 (b) Examine the economic factors that discourage entrepreneurial intentions. (10 marks)
 5(a) Explain the community threats that affect development in Uganda. (10 marks)
 (b) Suggest the possible solutions to the community threats in Uganda. (10 marks)
 6(a) Explain the beneficiaries of accounting information that is prepared within a business. (10 marks)
 (b) Identify the significance of a systematic financial recording of all business transactions. (10 marks)
 7(a) Examine the circumstances under which communication may be effective. (10 marks)
 (b) Explain the factors considered when selecting the channels of communication. (10 marks)

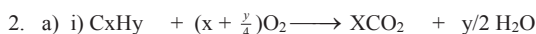
Find answers in New Vision next Tuesday

CHEMISTRY PAPER 1 SET 3 ANSWERS



c) i) $E_{\text{cell}} = E_{\text{right}} - E_{\text{left}}$
 $= 0.76 - 0.54$
 $= +0.22\text{V}$

ii) Feasible, because the e.m.f of the cell is positive.



ii) volume of CO_2 formed = $50 - 30 = 20\text{cm}^3$

volume of O_2 used = $60 - 30 = 30\text{cm}^3$

$10x = 20$

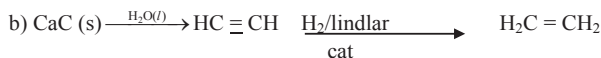
$x = 2$

$10(x + y/4) = 30$

$10(2 + y/4) = 30$

$y = 4$

the molecular formula of Q is C_2H_4



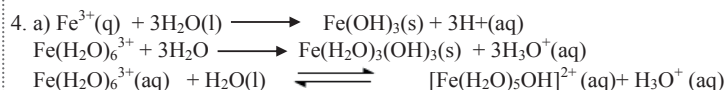
ii) $K_h = \frac{[\text{C}_6\text{H}_5\text{COOH}][\text{OH}^-]}{[\text{C}_6\text{H}_5\text{COO}^-]}$

b) $\text{pH} = -\log[\text{H}^+] = 8.6$

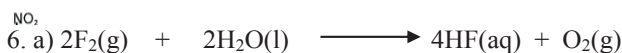
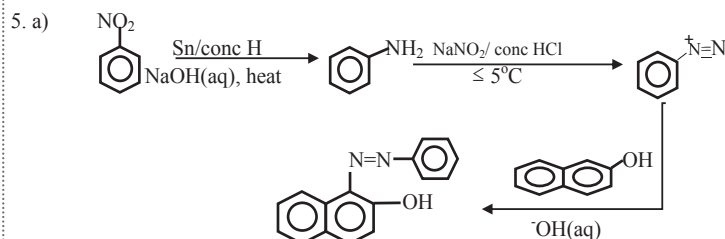
$[\text{H}^+] = 10^{-8.6} = 2.51 \times 10^{-9}\text{M}$

$[\text{OH}^-] = \frac{K_w}{[\text{H}^+]} = \frac{1 \times 10^{-14}}{2.5 \times 10^{-9}} = 3.98 \times 10^{-6}$

$K_h = \frac{[\text{H}^+]^2}{[\text{CH}_3\text{COO}^-]} = \frac{(12)}{44} \frac{(3.98 \times 10^{-6})^2}{0.2} = 1.58 \times 10^{-10} \text{mol dm}^{-3}$



b) i) brown precipitate and effervescence/bubbles of a colourless gas



8. a)

	CH ₃ CHCH ₂ CH ₂ CH ₃ OH	Reagent	
	Forms cloudy solution within 5 minutes	anhydrous zinc chloride and concentrated hydrochloric acid (lucas reagent)	Forms cloudy solution immediately
or	Purple solution turns colourless	Acidified potassium manganate (VII) solution	No observable change
b)	CH ₃ COO ⁻ Na ⁺	Acidified potassium manganate (VII) solution and heat	Purple solution turned colourless and bubbles of colourless gas given out.
or	Reddish brown solution	Iron (III) chloride solution	Light green solution

7. a) let x be extracted by ether, mass that remains in water = $(10-x)\text{g}$

$= \frac{(10-x)/200}{x/100} = 0.2$

$= 1.4x = 10$

$x = 7.14\text{g}$

b) let y g be extracted

$= \frac{(10-y)/200}{y/50} = 0.2$

$= 1.8y = 10$

$y = 5.56\text{g}$

Remainder = $10 - 5.56 = 4.44\text{g}$

let w g be extracted

$\frac{4.44 - w/200}{w/50} = 0.2$

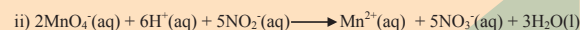
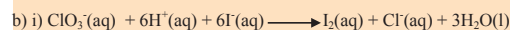
$1.8w = 4.44$

$w = 2.47\text{g}$

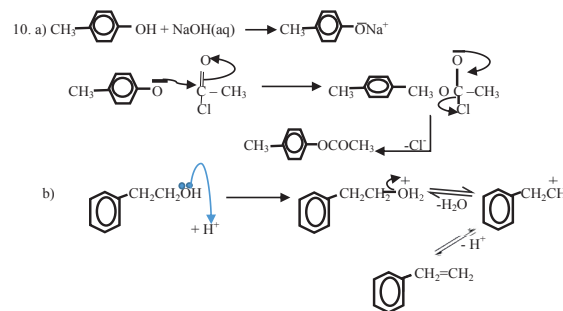
Total mass extracted = $5.56 + 2.47 = 8.03\text{g}$

9. a)

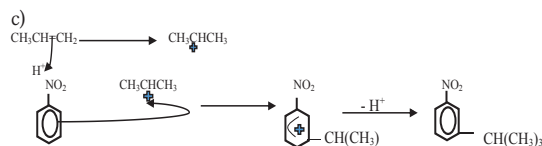
	Square pyramidal
	Trigonal pyramidal
	V - shape or bent



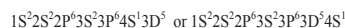
SECTION B



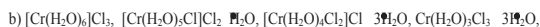
Turn to next page



11. a) i) write the electronic configuration of chromium atom



ii) Because it has partially filled d-orbitals



c) i) Grey green ppt, soluble in excess giving a purple or violet solution



12. Mass of carbon = $\frac{(12)}{44} \times 17.6 = 4.8\text{g}$

Mass of oxygen = $6.8 - (4.8 + 0.4) = 1.6\text{g}$

C H O

$\frac{4.8}{12} \quad \frac{0.4}{1} \quad \frac{1.6}{16}$

$\frac{0.4}{0.1} \quad \frac{0.4}{0.4} \quad \frac{0.1}{0.1}$

$4 \quad 4 \quad 1$

Empirical is $\text{C}_4\text{H}_4\text{O}$

b) $(\text{C}_4\text{H}_4\text{O})_n = 68 \times 2 = 136$

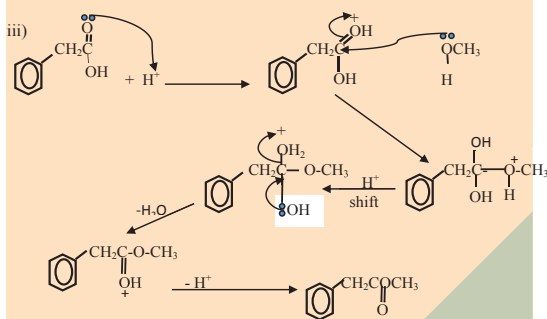
$48n + 4n + 16n = 136$

$68n = 136$

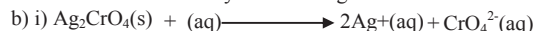
$n = 2$

Molecular formula = $\text{C}_8\text{H}_8\text{O}_2$

c) i)



13. a) is the precipitation of a sparingly soluble electrolyte from its saturated solution by adding a solution of amore soluble electrolyte containing one of its ions.



ii) $K_{sp} = [\text{Ag}^+]^2[\text{CrO}_4^{2-}]$

c) Rmm of $\text{Ag}_2\text{CrO}_4 = (2 \times 108) + 52 + (16 \times 4) = 332$

Solubility (s) = $3.21/332 = 9.67 \times 10^{-3}$

$K_{sp} = (2s)^2 s = 4s^3 = 4 \times (9.67 \times 10^{-3})^3$

$= 3.62 \times 10^{-6} \text{mol dm}^{-3}$

d) $[\text{Ag}^+]^2 \times 0.002 = 3.62 \times 10^{-6}$

$[\text{Ag}^+] = \sqrt{\frac{3.62 \times 10^{-6}}{0.002}} = 0.0425\text{M}$

CHEMISTRY SET 3 ANSWERS

e) $[\text{Ag}^+] \times 0.002 = 1.96 \times 10^{-10}$

$[\text{Ag}^+] = 9.8 \times 10^{-8} \text{mol dm}^{-3}$

Silver chloride will be precipitated first because the concentration of silver ions required to precipitate 0.002M chloride ions is lower than that required to precipitate 0.002M chromate ions.

14. i) Heating tin in dry chlorine

ii) Heating tin in dry hydrogen chloride

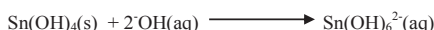
b) i) white solid/precipitate and white fumes



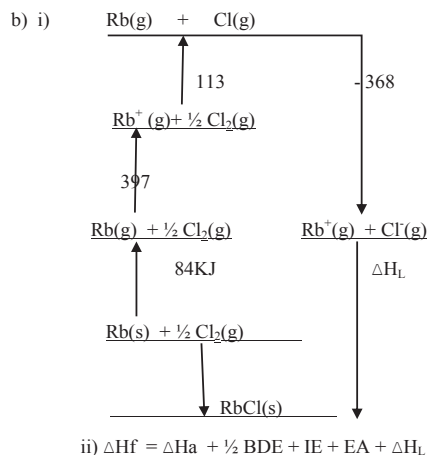
ii) cloudy solution is formed



c) i) white ppt soluble in excess giving a colourless solution



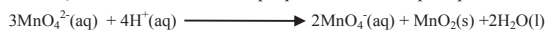
15. a) It is the energy change when one mole of ionic compound is formed from its constituent gaseous ions.



ii) **Charge on the ions:** the higher the charge on oppositely charged ions, the greater the attraction and the higher lattice energy.

Ionic radius: the smaller the ionic radius of the ions, the closer they can be and the stronger the ionic bond and hence the higher the lattice energy.

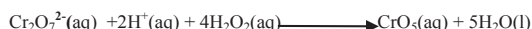
16. a) Green solution turn to purple and a brown precipitate



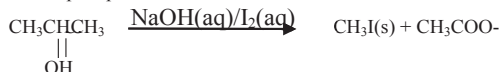
b) Reddish brown precipitate



c) Orange solution turned to an intense blue solution



d) Yellow precipitate



17. a) i) Thermoplastic – a linear polymer which melts or softens when heated and can be remoulded.

Thermosetting plastic – is a cross-linked linked polymers which does not melt or soften when heated and cannot be remoulded.

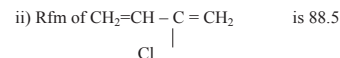
ii) Thermoplastic – polythene, nylon
Thermosetting – Bakelite, melamine



ii) Addition polymerisation

c) i) $\pi v = nRT = \frac{m}{M_r} \cdot R \cdot T$

$M_r = \frac{mRT}{\pi v} = \frac{2.5 \times 8.31 \times 298}{4.0 \times 10^{-4} \times 101325 \times 10^{-3}} = 152749.8$



$= \frac{152749.8}{88.5}$

$= 1726$

THE TEACHERS



MOSES MUGOGO,
SEETA HIGH SCHOOL



ANDREW HANNINGTON NSERENO
BISHOP'S SENIOR SCHOOL - MUKONO

CHEMISTRY PAPER 2 QUESTIONS ACHEMO4

SECTION A

- (a) Write the electronic configuration of manganese (atomic number of Mn = 25)

(b) Explain why manganese is

 - a transition element
 - has variable oxidation state
 - has a higher melting point (1890°C) than calcium with melting point (860°C)

(c) Describe the reaction of manganese with:

 - water
 - sulphuric acid

(d) Aqueous sodium hydroxide was added dropwise until in excess to an aqueous solution of manganese(II) sulphate.

 - State what was observed
 - Write equation (s) for the reaction (s) that took place
 - Write an equation for the reaction that takes place between trimanganesetetroxide and aluminium

2. The table below shows the variation in pH when 30cm³ of 0.2M ammonia solution was titrated with hydrochloric acid:

Volume of HCl added/cm ³	0	4	8	12	16	18	19
PH	10.8	9.9	9.4	9.1	8.7	8.3	8.0

Volume of HCl added/cm ³	19.4	19.8	20.2	20.6	21	22	26	28
PH	7.8	7.3	3.9	3.5	3.2	2.9	2.5	2.4

- Plot a graph of pH against volume of HCl added
- Use the graph to determine the;
 - pH and volume at the end point
 - Molarity of hydrochloric acid solution
 - Hydrolysis constant of ammonium chloride formed at the end point
 - Ratio of $[\text{NH}_4\text{Cl}]:[\text{NH}_3]$ when 10cm³ of HCl has been added to ammonia solution (K_b for ammonia = $1.78 \times 10^{-5} \text{mol dm}^{-3}$, $K_w = 1 \times 10^{-14} \text{mol}^2 \text{dm}^{-6}$)
- Explain the shape of the graph
- Which of the indicators shown below is suitable for the titration? Give a reason for your answer.

Indicator	pH range
Thymol blue	1.2-2.8
Methylorange	3.1-4.4
Methy red	4.2-6.3
Phenolphthalein	8.3-10.0

3. When 7.5g of an organic compound Q was burnt completely in excess oxygen 11.2dm³ of carbon dioxide and 4.5g of water were formed at s.t.p.

- (i) Calculate the empirical formula of Q
- (ii) Determine the molecular formula of Q (Density of Q is 5.357gdm⁻³ at s.t.p)
- (iii) Q burns with a sooty flame and forms a yellow precipitate with 2,4-dinitrophenylhydrazine and also forms a pale yellow precipitate with iodine solution in sodium hydroxide solution. Identify Q
- (b) Write equation and suggest a mechanism for the reaction between Q and;
 - (i) 2, 4-dinitrophenylhydrazine in acidic medium
 - (ii) Sodiumhydrogensulphite solution
- (c) Using equations only show how Q;
 - (i) Can be synthesized from benzaldehyde
 - (ii) Can be converted to an alkene

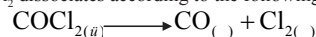
- (a) (i) State Le Chaterier's principle
- (ii) State two factors that affect equilibrium reactions apart from catalyst
- (iii) Briefly describe how each of the factors you have named in (a)(i) affect the equilibrium constant and equilibrium position.
- (b) Given the reaction;



Write the expression for the equilibrium constant K_c and K_p giving units in each case.

CHEMISTRY PAPER 2 QUESTIONS (ACHEM04)

What is the effect on equilibrium position of adding a catalyst
(c) COCl_2 dissociates according to the following equation;



- (i) At 25° C, one mole of COCl_2 was placed in 2dm³ vessel producing an equilibrium mixture with 20.25% chlorine. Calculate the value of the equilibrium constant K_c at this temperature.
- (ii) At 75° C, the degree of dissociation of 2 moles of COCl_2 in the same 2dm³ vessel was found to be 15%. Calculate the value of the equilibrium constant K_c at this temperature.
- (iii) From your answer in(c) (i) and (c)(ii) above, state whether the reaction is exothermic. Give a reason for your answer.

SECTION B

5. Write equation to show how the following conversion can be effected.

- (a) Bromo benzene from phenol
- (b) Propanal from chloroethane
- (c) Phenyl amine from methylbenzene
- (d) Hexane from propene
- (e) $(\text{CH}_3)_2\text{NCH}_2\text{CH}_2\text{CH}_3$ from 2-iodopropane

6. (a) Define the following terms

- (i) Lattice energy
- (ii) Hydration energy
- (b) Given the following data;
Standard enthalpy of formation of aluminium fluoride = -1301 KJ/ mole
Enthalpy of atomization of aluminium = 314 KJ/ mole

Bond dissociation energy of fluorine = 158 KJ/ mole
First ionization energy of aluminium = 577 KJ/ mole
Second ionization energy of aluminium = 1820 KJ/mole
Third ionization energy of aluminium = 2740 KJ / mole
First electron affinity of fluorine = -348 KJ / mole

- (i) Draw an energy level diagram for the formation of aluminium fluoride and use it to determine the lattice energy of aluminium fluoride
- (ii) If the hydration energies of aluminium ions and fluoride ions are -4690 and -364 KJ/mole respectively, calculate the enthalpy of solution of aluminium fluoride and hence comment on the solubility of aluminium fluoride
- (d) (i) State and explain two factors that affect lattice energy.
- (e) Would you expect lattice energy of aluminium chloride to be less than, equal to, or greater than that of aluminium fluoride? Explain your answer.

7. (a) Describe the industrial preparation of chlorine.

(b) Describe the reaction of chlorine with

- (i) Iron
- (ii) Potassium hydroxide
- (c) Describe the reaction of fluorine with:
 - (i) Water
 - (ii) Potassium hydroxide
- (d) Explain why hydrogen fluoride is a liquid whereas hydrogen chloride is a gas at 18°C.

8. (a) Explain the following processes as used in the extraction of metals:

- (i) Floatation
- (ii) Roasting
- (iii) Smelting

(b) Briefly describe how the ore of aluminium can be concentrated

**Find answers only in
New Vision next Tuesday**

MATHEMATICS QUESTIONS FOR SOLUTIONS PUBLISHED LAST TUESDAY

1. Event A and B are such that

$$P(A) = \frac{8}{15}, P(B) = \frac{1}{3} \text{ and } P(A/B) = \frac{1}{5}. \text{ Find the}$$

probability that;

- (i) neither A nor B occurs.
 - (ii) event B does not occur if event A has occurred.
2. A bullet of mass 150g moving at a speed of 216kmh⁻¹ penetrates 10cm into a fixed wooden rectangular block before coming to rest. Find the;
- (a) Resistance due to the block.
 - (b) Velocity of the bullet when it penetrates 4cm into the block.
3. The resistance of a wire at different temperatures is as follows.

Resistance (Ω)	14	32
Temperature (°C)	5	41

Use linear interpolation or extrapolation to estimate the:

- (i) Temperature corresponding to 35Ω
 - (ii) Resistance whose value is equal to that of the temperature.
4. A particle executes simple harmonic motion (S.H.M) about centre O, with amplitude 6m and frequency $\frac{2}{\pi}$ Hertz. Find the distance from O at which its speed is half the maximum speed.
5. Use trapezium rule with 7 ordinates to find the value of $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sin^2\left(\frac{x}{2}\right) dx$ correct to 2 decimal places.
6. A random variable x has a probability density function given by
- $$f(x) = \begin{cases} \frac{2}{3}x, & 0 \leq x \leq 1 \\ \frac{1}{3}(3-x), & 1 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$
- Calculate the median of x.
7. The following grades were obtained by 8 candidates in mathematics and in general paper.

THE TEACHERS



IVAN GIMEJ,
ST MARY'S COLLEGE, KISUBI



AUGUSTUS ISINGOMA,
ST JOHN'S SS, NYABWINA - SHEEMA

Mathematics	A	O	B	F	E	C	D	B
General paper	C	D	D	P	P	D	C	D
	3	2	1	8	8	2	3	2

Calculate the rank correlation coefficient for the grades and comment on your result at 5% level of significance.

8. ABC is an Isosceles triangle, right angled at A. Forces of 3N, 4N and 6N along the sides BA, CB and CA respectively. Find the magnitude and direction of the resultant force.

SECTION B

- (a) A ball is served horizontally from a height of 2m, 12m away from a net 1.2m high. If the ball just clears the net, find the speed with which the ball is projected.
- (b) A body moving with acceleration $e^{2t}i - 3\sin 2tj + 4\cos 2tk$ is initially located at the point (1, -2, 2)m and has velocity of $4i - 2j + k \text{ ms}^{-1}$. Find the;
 - (i) Speed of the body when $t = \frac{\pi}{4} \text{ s}$.

- (ii) Distance of the body from the origin $t = \frac{\pi}{4} \text{ s}$.

10. The table below shows the weight (in kg) of patients who visited the hospital in a certain week.

Weight (kg)	0-<20	20-<30	30-<40	40-<50	50-<60	60-<70	70-<80
Number of patients	30	16	24	32	28	12	2

Draw an origin and use it to estimate the

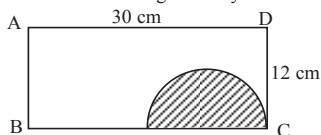
- (i) Probability that a patient weighing between 13kg and 52kg visited the hospital.
 - (ii) The middle 60% range of the weight of patients.
 - (iii) The weight exceeded by the 5% of the students.
11. a) Show graphically that the equation $xe^x = x + 1$ has a root in the interval 0 and 1.
b) Using the Newton's Raphson Method, find the root, correct to 3 decimal places.
12. A particle A of weight 49N is in contact with a horizontal table connected by a light inelastic string passing over a smooth light pulley fixed at the edge of the table. The other end of the string carries another particle B of mass 2kg hanging freely. The system is released from rest and after 2 seconds A collided and coalesced with a stationary particle of mass 0.1kg at rest on the table. If the coefficient of friction between the table and the weight is 0.25.
- (a) Calculate the;
 - (i) Acceleration of the system.
 - (ii) Tension in the string before collision.
 - (b) Find the change in kinetic energy of A immediately after impact.
13. a) Points A and B are 25.5km apart. A body is timed as it moved from A to B and it takes 0.426 hours. Determine
(i) The maximum possible error in distance and time.

Turn to next page

MATHEMATICS QUESTIONS FOR SOLUTIONS PUBLISHED LAST TUESDAY

- (ii) The range within which the calculated s_j lies.
- b) The radius r and height h of a cylinder are measured with corresponding possible errors Δr and Δh .
- (i) Show that the maximum possible relative error in volume is $\left| \frac{\Delta h}{h} \right| + 2 \left| \frac{\Delta r}{r} \right|$.
- (ii) If h and r are measured with percentage errors of 2% and 1.4% respectively. Find the corresponding percentage error in volume.
14. a) A random sample of 36 chicken from a poultry farm is taken. These have mass (measured in kg) of x_i such that
- $$\sum_{i=1}^{36} x_i = 144.9 \text{ and } \sum_{i=1}^{36} x_i^2 = 588.69$$
- Calculate a 95% confidence interval for the mean.
- b) The weights of ball bearing are normally distinguished with mean 25g and standard deviation 4grams. If a random sample of 16 ball bearings is taken, find the probability that the mean of the sample lies between 24.12 grams and 26.73 grams.
15. a) In S.6 class, 40% of the candidates support David for the post of head prefect. If a random sample of 10 candidates is selected. Find the probability that more than 5 candidates support David for the post.
- b) An examination has two parts A and B, the probability of a student getting part A correct is $\frac{2}{3}$. If she gets part A correct, the probability that she/he gets B correct is $\frac{3}{4}$, otherwise it is $\frac{1}{6}$. There are three marks for a correct solution of part A, two marks for a correct solution of part B and a bonus mark if both parts are correct.
- (i) Construct a probability distribution table for the total marks scored by the student.
- (ii) Find the expected student's total mark.

16. a) Show that the centre of mass of a uniform semi-circular lamina is $\frac{4r}{3\pi}$ from the bounding diameter.
- b) The figure below shows a rectangular piece of card board ABCD measuring 30cm by 12cm.



- If a semi-circle of radius 5cm is removed:
- (i) Calculate the coordinates of the centre of mass of the remaining piece of card board taking BC and AB as the X and Y axes respectively.
- (ii) Given that the remaining card is suspended from A and hangs in equilibrium, find the angle AD makes with the vertical.

SOLUTIONS TO PURE MATHEMATICS PUBLISHED ON MAY 12

SECTION A

1. $3\cos^2 x - 3\sin x \cos x + 2\sin^2 x = 1$
Dividing through by $\cos^2 x$
 $3 - 3\tan x + 2\tan^2 x = 1 + \tan^2 x$
 $\tan^2 x - 2\tan x + 2 = 0$
 $\tan^2 x - \tan x - 2\tan x + 2 = 0$
 $\tan x(\tan x - 1) - 2(\tan x - 1) = 0$
 $(\tan x - 2)(\tan x - 1) = 0$
 $\tan x = 2$ and $\tan x = 1$
 $x = \tan^{-1}(2)$ Also
 $x = 63.4^\circ$ 243.4° $x = \tan^{-1}(1)$
 $x = 45^\circ$ 225°
Therefore $x = 63.4^\circ$, 243.4° , 45° and 225° .

2. Consider L.H.S using $\int v \frac{dy}{dx} dx = uv - \int u \frac{dv}{dx} dx$

Let $v = \log_{10}^+ x$ $\int du = \int x dx$
 $10^v = x$ $u = \frac{x^2}{2} + c$
 $v \ln 10 = \ln x$

$$\frac{dv}{dx} = \frac{1}{x \ln 10}$$

$$\int_1^{10} x \log_{10}^+ x dx = \left[\frac{x^2}{2} \log_{10}^+ x \right]_1^{10} - \frac{1}{2 \ln 10} \int_1^{10} x dx$$

$$= 50 - \frac{1}{4 \ln 10} [x^2]_1^{10}$$

$$= 50 - \frac{1}{4 \ln 10} [100 - 1]$$

$$= 50 - \frac{99}{4 \ln 10}$$

3. $\left| \frac{z-1}{z+1} \right| = 2$

Let $z = x + iy$

$$\left| \frac{x+iy-1}{x+iy+1} \right| = 2$$

$$\left| \frac{(x-1)+iy}{(x+1)+iy} \right| = 2$$

$$\sqrt{(x-1)^2 + y^2} = 2\sqrt{(x+1)^2 + y^2}$$

$$x^2 - 2x + 1 + y^2 = 4[x^2 + 2x + 1 + y^2]$$

$$x^2 + y^2 - 2x + 1 = 4x^2 + 4y^2 + 8x + 4$$

$$3x^2 + 3y^2 + 10x + 3 = 0$$

$$x^2 + y^2 + \frac{10}{3}x + 1 = 0$$

The locus is a circle centre $(-\frac{10}{3}, 0)$ and radius $\frac{\sqrt{91}}{3}$.

4. $\alpha + \beta = 26$, $\alpha\beta = 10$

Sum of the roots

$$\beta\alpha^3 + \alpha\beta^3 = \alpha\beta(\alpha^2 + \beta^2)$$

$$= \alpha\beta((\alpha + \beta)^2 - 2\alpha\beta)$$

$$= 10[676 - 20]$$

$$= 6,560$$

Product of the roots $(\beta\alpha^3)(\alpha\beta^3)$

$$= \alpha^4\beta^4$$

$$= (\alpha\beta)^4$$

$$= (10)^4$$

$$= 10,000$$

$$x^2 - (\text{sum of the roots})x + \text{product of the roots} = 0$$

$$x^2 - 6560x + 10,000 = 0 \text{ is the required equation.}$$

5. $y = \ln \tan \frac{\theta}{2}$

$$\frac{dy}{d\theta} = \frac{\frac{1}{2} \sec^2 \frac{\theta}{2}}{\tan \frac{\theta}{2}}$$

$$= \frac{1}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}$$

$$\frac{dy}{d\theta} = \frac{1}{\sin \theta}$$

$$\frac{dy}{d\theta} = \csc \theta$$

$$\frac{d^2 y}{d\theta^2} = -\csc \theta \cot \theta$$

$$\frac{d^2 y}{d\theta^2} = -\cot \theta \frac{dy}{d\theta}$$

$$\frac{d^2 y}{d\theta^2} + \cot \theta \frac{dy}{d\theta} = 0$$

6. $d_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $d_2 = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$

$$\cos \theta = \frac{d_1 \cdot d_2}{|d_1| |d_2|}$$

$$= \frac{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}}{\left(\sqrt{1+4+9} \right) \left(\sqrt{4+9+16} \right)}$$

$$\cos \theta = \frac{20}{\sqrt{406}}$$

$$\theta = \cos^{-1} \left(\frac{20}{\sqrt{406}} \right)$$

$$\theta = 6.98^\circ \text{ (2dp's)}$$

7. Using the general equation of the circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

At (2, 3)

$$4 + 9 + 4g + 6f + c = 0$$

$$4g + 6f + c = -13 \dots\dots\dots (i)$$

At (3, 2)

$$9 + 4 + 6g + 4f + c = 0$$

$$6g + 4f + c = -13 \dots\dots\dots (ii)$$

At (6, 1)

$$36 + 1 + 12g + 2f + c = 0$$

$$12g + 2f + c = -37 \dots\dots\dots (iii)$$

Eliminating c in equation (i) and (ii) gives

$$-2g + 2f = 0$$

$$g = f \dots\dots\dots (iv)$$

Eliminating c in equation (i) and (iii) gives

$$-8g + 4f = 24$$

$$-2g + f = 6 \dots\dots\dots (v)$$

Substituting equation (iv) in equation (v) gives

$$-2g + g = 6$$

$$-g = 6$$

$$g = -6$$

From (iv) $f = -6$.

Substituting $f = -6$ and $g = -6$ in equation (1) to get c

$$-24 - 36 + c = -13$$

$$c = -13 + 60$$

$$c = 47$$

Substituting $g = -6$, $f = -6$ and $c = 47$ to the general equation of the circle gives

$$x^2 + y^2 - 12x - 12y + 47 = 0. \text{ Is the}$$

required equation of the circle.

8. $\frac{dy}{dx} + 2y = e^{3x} \dots\dots\dots (i)$

If $= e^{\int 2dx}$

$$= e^{2x} \dots\dots\dots (ii)$$

Eqn (i) $\times$ (ii) gives

$$e^{2x} \frac{dy}{dx} + 2e^{2x} y = e^{5x}$$

$$\frac{d}{dx} (e^{2x} y) = e^{5x}$$

$$\int \frac{d}{dx} (e^{2x} y) dx = \int e^{5x} dx$$

$$e^{2x} y = \frac{1}{5} e^{5x} + c \text{ for } x=0, y=\frac{5}{6}$$

$$\frac{5}{6} = \frac{1}{5} + c$$

$$c = \frac{19}{30}$$

$$e^{2x} y = \frac{1}{5} e^{5x} + \frac{19}{30}$$



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SOLUTIONS TO PURE MATHEMATICS PUBLISHED ON MAY 12

$$y = \frac{e^{3x}}{5} + \frac{19e^{-2x}}{30}$$

SECTION B

9. a) Let $P = \bar{Z} = \frac{3}{Z}$

$$P = \frac{3+2i}{1} - \frac{3}{3-2i}$$

$$P = \frac{(3+2i)(3-2i)-3}{3-2i}$$

$$P = \frac{9-(2i)^2-3}{3-2i}$$

$$P = \frac{13-3}{3-2i} \quad \text{Since } i^2 = -1$$

$$P = \frac{10}{(3-2i)(3+2i)}$$

$$P = \frac{30}{13} + \frac{20i}{13}$$

Therefore, $\bar{Z} = \frac{3}{z} = \frac{30}{13} + \frac{20}{13}i$

$$|P| = \sqrt{\left(\frac{30}{13}\right)^2 + \left(\frac{20}{13}\right)^2}$$

$$= \frac{\sqrt{1300}}{13}$$

$$= \frac{10\sqrt{13}}{13}$$

Therefore $\left|\bar{Z} - \frac{3}{z}\right| \square 2.7735 \quad (4\text{dp's})$

$$\text{Arg } P = \tan^{-1}\left(\frac{20}{30}\right)$$

$$= \tan^{-1}\left(\frac{2}{3}\right)$$

$$= 33.69^\circ$$

Therefore $\text{Arg}\left(\bar{Z} - \frac{3}{z}\right) \square 33.7^\circ \quad (1\text{dp})$

b) Let $z = x + iy$

$$\left|\frac{3(x+iy)+1}{2(x+iy)-i}\right| = \sqrt{2}$$

$$\left|\frac{(3x+1)+3iy}{2x+(2y-1)i}\right| = \sqrt{2}$$

$$\sqrt{(3x+1)^2 + (3y)^2} = \sqrt{2}\sqrt{(2x)^2 + (2y-1)^2}$$

$$9x^2 + 6x + 1 + 9y^2 = 2(4x^2 + 4y^2 - 4y + 1)$$

$$9x^2 + 9y^2 + 6x + 1 = 8x^2 + 8y^2 - 8y + 2$$

$$x^2 + y^2 + 6x + 8y - 1 = 0$$

By completing squares

$$(x+3)^2 - 9 + (y+4)^2 - 16 - 1 = 0$$

$$(x+3)^2 + (y+4)^2 = 26$$

The locus is a circle centre $(-3, 4)$ and radius

$$r = \sqrt{26}$$

10. (a) $\frac{x^3-1}{(x+3)(x^2+1)} = \frac{x^3-1}{x^3+3x^2+x+3}$

Using division

$$\frac{x^3+3x^2+x+3}{x^3+3x^2+x+3}$$

$$= 1 + \frac{-(3x^2+x+4)}{(x+3)(x^2+1)}$$

Consider $\frac{3x^2+x+4}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1}$

$$3x^2+x+4 = A(x^2+1) + (Bx+C)(x+3)$$

For $x = -3$

$$28 = 10A + 0$$

$$A = \frac{14}{5}$$

For $x = 0$

$$4 = \frac{14}{5} + 3C$$

$$C = \frac{2}{5}$$

For $x = 1$

$$8 = \frac{28}{5} + 4B + \frac{8}{5}$$

$$B = \frac{1}{5}$$

$$\frac{3x^2+4x+4}{(x+3)(x^2+1)} = \frac{14}{5(x+3)} + \frac{x+2}{5(x^2+1)}$$

Substituting equation (ii) in (i) gives

$$\frac{x^3-1}{(x+3)(x^2+1)} = 1 - \frac{14}{5(x+3)} - \frac{x}{5(x^2+1)} - \frac{2}{5(x^2+1)}$$

b)

$$\int \frac{x^3-1}{(x+3)(x^2+1)} dx = \int dx - \frac{14}{5} \int \frac{1}{x+3} dx - \frac{1}{5} \int \frac{x}{x^2+1} dx - \frac{2}{5} \int \frac{1}{x^2+1} dx$$

$$x - \frac{14}{5} \ln|x+3| - \frac{1}{10} \ln(x^2+1) - \frac{2}{5} \tan^{-1}x + c$$

(a) Sketch the curve $y = \frac{x^2+2}{(x-2)(x+2)}$ stating clearly

the asymptotes.

Solution

$$y = \frac{x^2+2}{x^2-4}$$

$$yx^2 - 4y = x^2 + 2$$

$$(y-1)x^2 - 4y - 2 = 0$$

For real roots

$$b^2 - 4ac \geq 0$$

$$0 - 4(y-1)(-4y-2) \geq 0$$

$$(y-1)(4y+2) \geq 0$$

The critical values are $y = 1$ and $y = -\frac{1}{2}$

	$y < -\frac{1}{2}$	$-\frac{1}{2} < y < 1$	$y > 1$
$(y-1)(4y+2)$	+	-	+

$y < -\frac{1}{2}$ or $y > 1$ from the table.

So $-\frac{1}{2} < y < 1$

$$\text{For } (y-1)(4y+2) \geq 0$$

$$y \leq -\frac{1}{2} \text{ or } y \geq 1$$

For $y = 1$

$$\frac{x^2+2}{x^2-4} = 1$$

$$x^2+2 = x^2-4$$

$$2 = -4 \text{ (impossible) ignore it}$$

Either $y = -\frac{1}{2}$ or $y > 1$

$$\frac{-1}{2} \geq \frac{x^2+2}{x^2-4} \text{ is the region.}$$

At $y = 1$, the turning point does not exist for $y = \frac{1}{2}$

the curve is maximum.

$$\frac{-1}{2} = \frac{x^2+2}{x^2-4}$$

$$2x^2+4 = -x^2+4$$

$$3x^2 = 0$$

$$x = 0$$

$(0, \frac{1}{2})$ is the maximum point

Vertical asymptotes $x^2-4 = 0$
 $x = \pm 2$.

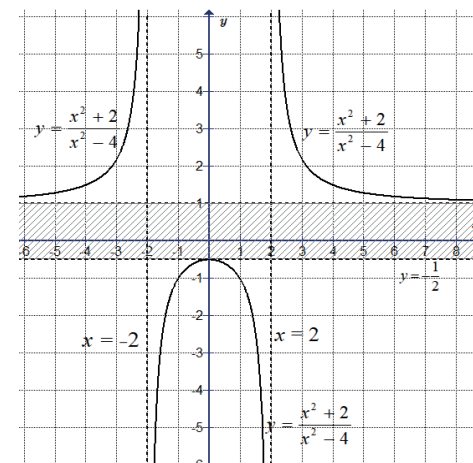
$$\text{As } x \rightarrow \pm \infty, \text{ for } y = \frac{1+\frac{2}{x}}{1-\frac{1}{x^2}}$$

$$y \rightarrow 1$$

$y = 1$ is the horizontal asymptote.

Critical values are $x = \pm 2$

	$x < -2$	$-2 < x < 2$	$x > 2$
x^2+2	+	+	+
x^2-4	+	-	+
y	+	-	+



12. (a) (i) $\frac{x+1}{2} = \frac{y-3}{5} = \frac{z+1}{-1} = k$

$$\begin{cases} x = 2k - 1 \\ y = 5k + 3 \\ z = -k - 1 \end{cases} \rightarrow (I)$$

Substituting equation (I) to the equation of the plane $2x - y + 3z = 20$ gives

$$2(2k-1) - (5k+3) + 3(-k-1) = 20$$

$$4k - 2 - 5k - 3 - 3k - 3 = 20$$

$$-4k = 28$$

$$k = -7$$

Substituting $k = -7$ in equation (I) gives $x = -15$, $y = -32$ and $z = 6$

$\therefore$ Point of intersection is $(-15, -32, 6)$.

(ii) Using $\sin \theta = \frac{n \cdot d}{|n| |d|}$ where $n = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

$$d = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$$

$$\sin \theta = \frac{\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}}{\sqrt{(2)^2 + (-1)^2 + (3)^2} \sqrt{(2)^2 + (5)^2 + (-1)^2}}$$

$$\sin \theta = \frac{-4}{\sqrt{14} \times \sqrt{30}}$$

$$\theta = \sin^{-1}\left(\frac{-4}{\sqrt{420}}\right)$$

$$\theta = 11.3^\circ$$

(b) $\overrightarrow{AP} = \frac{2}{5} \left[\begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -7 \\ 9 \end{pmatrix} \right] = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$

$$= \begin{pmatrix} -7 \\ +9 \end{pmatrix} + \begin{pmatrix} -4 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} -3 \\ 7 \end{pmatrix}$$

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$$\overrightarrow{CQ} = \frac{1}{-3} \left[\begin{pmatrix} -2 \\ -9 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right]$$

$$= \begin{pmatrix} 1 \\ 13 \end{pmatrix}$$

$$\overrightarrow{OQ} = \overrightarrow{OC} + \overrightarrow{CQ}$$

$$= \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 13 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 15 \end{pmatrix}$$

$$r = a + \lambda d$$

$$\text{where } a = \begin{pmatrix} -3 \\ 7 \end{pmatrix} \text{ and } d = \begin{pmatrix} 2 \\ 15 \end{pmatrix} - \begin{pmatrix} -3 \\ 7 \end{pmatrix}$$

$$\therefore d = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

$$r = \begin{pmatrix} -3 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 8 \end{pmatrix} \text{ is the required vector equation of the line.}$$

13. (a) (i) Let $y = x^x \cos x$

$$\ln y = \ln x^x + \ln \cos x$$

$$\ln y = x \ln x + \ln \cos x$$

$$\frac{1}{y} dy = \left[x \cdot \frac{1}{x} + \ln x + \frac{-3 \sin x}{\cos x} \right] dx$$

$$\frac{dy}{dx} = [1 + \ln x - 3 \tan x] \times (y)$$

$$\therefore \frac{dy}{dx} = x^x \cos 3x (1 + \ln x - 3 \tan 3x)$$

(ii) Let $y = 2^x \ln(2x - 5)$

Using product rule

$$\frac{dy}{dx} = 2^x \left(\frac{2}{2x-5} \right) + [\ln(2x-5)] 2^x \ln 2$$

$$= 2^x \left(\frac{2}{2x-5} + [\ln(2x-5)] \ln 2 \right)$$

(iii) $y = 3x^3 - 5x^2 + 2x$

$$\frac{dy}{dx} = 9x^2 - 10x + 2$$

At (1, 1)

$$\frac{dy}{dx} = 1. \text{ (the gradient of the tangent)}$$

$$\frac{y-1}{x-1} = 1$$

$$y-1 = x-1$$

$y = x$ is the equation of the tangent.

Using $M_1 M_2 = -1$

$M_2 = -1$ is the gradient of the normal.

$$\frac{y-1}{x-1} = -1$$

$$y = -x + 1 + 1$$

$y = -x + 2$ is the equation of the normal

14. LHS: $\cos 4x = 2\cos^2 3x - 1$

$$2[2\cos^2 x - 1] - 1$$

$$2[4\cos^4 x - 4\cos^2 x + 1] - 1$$

$$8\cos^4 x - 8\cos^2 x + 1$$

$$\frac{8}{\sec^4 x} - \frac{8}{\sec^2 x} + 1$$

$$\frac{8 - 8\sec^2 x + \sec^4 x}{\sec^4 x}$$

$$\frac{8 - 8 - 8\tan^2 x + 1 + 2\tan^2 x + \tan^4 x}{1 + 2\tan^2 x + \tan^4 x}$$

$$\frac{\tan^4 x - 6\tan^2 x + 1}{\tan^4 x + 2\tan^2 x + 1}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

b) $3(2\sin x \cos x) + 4\cos^2 x = -1$

$$6\sin x \cos x + 4\cos^2 x = -1$$

Dividing both sides by $\cos^2 x$

$$6\tan x + 4 = -(1 + \tan^2 x)$$

$$\tan^2 x + 6\tan x + 5 = 0$$

$$\tan^2 x + \tan x + 5\tan x + 5 = 0$$

$$\tan x(\tan x + 1) + 5(\tan x + 1) = 0$$

$$(\tan x + 5)(\tan x + 1) = 0$$

$$\tan x = -5 \text{ and } \tan x = -1$$

$$x = \tan^{-1}(-5)$$

$$x = 101.3^\circ$$

$$x = \tan^{-1}(-1)$$

$$x = 135^\circ$$

$$\therefore x = 101.3^\circ \text{ and } 135^\circ$$

15. a) $x = 4\sin \theta$

$$\cos \theta = \frac{x}{4} \dots\dots\dots (i)$$

$$y = 3\sin \theta$$

$$\sin \theta = \frac{y}{3} \dots\dots\dots (ii)$$

Using $\cos^2 \theta + \sin^2 \theta = 1$

$$\left(\frac{x}{4} \right)^2 + \left(\frac{y}{3} \right)^2 = 1$$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1 \text{ (Ellipse)}$$

Comparing the equation $\frac{x^2}{16} + \frac{y^2}{9} = 1$ with the standard

equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

$$a^2 = 16 \quad b^2 = 9$$

$$\text{and for the ellipse } b^2 = a^2(1 - e^2)$$

$$9 = 16(1 - e^2) \text{ Where } e \text{ is the eccentricity}$$

$$e^2 = \frac{7}{16}$$

$$e = \frac{\pm\sqrt{7}}{4}$$

b) Substituting $y = mx + c$ in the equation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} + \frac{(mx + c)^2}{b^2} = 1$$

$$b^2 x^2 + a^2(m^2 x^2 + 2mcx + c^2) = a^2 b^2$$

$$(b^2 + a^2 m^2)x^2 + 2a^2 mcx + a^2(c^2 - b^2) = 0$$

For tangency $b^2 = 4ac$

$$(2a^2 mc)^2 = 4(b^2 + a^2 m^2)a^2(c^2 - b^2)$$

$$4a^4 m^2 c^2 = 4a^2(b^2 + a^2 m^2)(c^2 - b^2)$$

$$a^2 m^2 c^2 = b^2 c^2 - b^2 + a^2 m^2 c^2 - a^2 m^2 b^2$$

$$b^2 c^2 = b^4 + a^2 m^2 b^2$$

Dividing by b^2 gives

$$c^2 = b^2 + a^2 m^2$$

$$c^2 = a^2 m^2 + b^2 \text{ as required.}$$

$$c = \pm\sqrt{a^2 m^2 + b^2} \quad a^2 = 16$$

$$b^2 = 9$$

Equation of the tangent must be in the form $y = mx + c$

$$y = mx \pm \sqrt{16m^2 + 9}$$

At (-3, 3)

$$3 = -3m \pm \sqrt{16m^2 + 9}$$

$$(3 + 3m)^2 = 16m^2 + 9$$

$$9 + 18m + 9m^2 = 16m^2 + 9$$

$$7m^2 - 18m = 0$$

$$m(7m - 18) = 0$$

$$m = 0, m = \frac{18}{7}$$

for $m = 0$

$$c = \pm 3$$

$$y = \pm 3$$

$$\text{For } m = \frac{18}{7}, c = \pm\sqrt{16\left(\frac{18}{7}\right)^2 + 9}$$

$$= \pm \frac{75}{7}$$

$$y = \frac{18}{7}x \pm \frac{75}{7}$$

$\therefore$ The equation of the tangent at the point (-3, 3) are

$$y = \pm 3 \text{ and } y = \frac{18x}{7} \pm \frac{75}{7}$$

16. a) $\frac{dy}{dx} = y \tan x = \cos x \dots\dots\dots (i)$

$$\text{If } = e^{\int \tan x dx}$$

$$\text{If } = e^{\ln \cos x}$$

$$= \cos x \dots\dots\dots (ii)$$

(i) $\times$ (ii)

$$\cos x \frac{dy}{dx} - y \sin x = \cos^2 x$$

$$\frac{d}{dx}(y \cos x) = \cos^2 x$$

$$\int \frac{d}{dx}(y \cos x) dx = \int \cos^2 x dx$$

$$y \cos x = \frac{1}{2} \int (1 + \cos 2x) dx$$

$$y \cos x = \frac{1}{2} \left(x + \frac{1}{2} \sin x \right) + c$$

$$\frac{d\theta}{dt} \propto (\theta - 20)$$

$$\frac{d\theta}{dt} = -K(\theta - 20)$$

$$\int \frac{d\theta}{\theta - 20} = -K \int dt$$

$$\ln(\theta - 20) = -kt + c$$

At $t = 0, \theta = 100^\circ\text{C}$

$$c = \ln 80$$

$$\ln(\theta - 20) = -Kt + \ln 80$$

$$\text{At } t = 20 \text{ minutes, } \theta = 60^\circ$$

$$\ln 40 - \ln 80 = -20K$$

$$\ln\left(\frac{1}{2}\right) = -20K$$

$$\ln 2 = 20K$$

$$K = \frac{1}{20} \ln 2$$

$$\ln(30 - 20) = \frac{1}{20} \ln 2 + \ln 80$$

$$t \frac{\ln 2}{20} = \ln 80 - \ln 10$$

$$t \frac{\ln 2}{20} = \ln 8$$

$$t = 60 \text{ minutes}$$

Find more questions in
New Vision next Tuesday



Write
to us:

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